CHAPTER 1

Philosophy and Logic of Physical Theory

We have undertaken to describe and study a particular theory in physics. It is therefore necessary that we ask ourselves exactly what this entails. Is it simply a set of equations from which other equations are derived? And if it were so, where do the first equations come from? Are we doomed from the start, because we are thinking about a classical theory when we know that classical physics is "wrong" and quantum physics is "right"? To what extent does it make sense to talk about an electron, say, in classical terms? These and similar questions clearly indicate that ignoring philosophy in physics means not understanding physics. For there is no theoretical physics without some philosophy; not admitting this fact would be self-deception. Let us then start with a very brief excursion into the philosophy of theoretical physics.

1-1 THE NATURE AND AIM OF THEORY IN PHYSICS

It is commonplace to say that physics is an experimental science. It is usually admitted that theory is at least useful. It is not always appreciated that theory is in general necessary in order to carry out meaningful experiments.

One could attempt to define theoretical physics as an intellectual enterprise in which one tries to bring order into the unending variety of physical phenomena by means of a logical-mathematical scheme. But this "scheme" is obviously of a very special kind. While one often speaks of "theory" in a very vague fashion, something quite specific emerges when the study of a particular subject matter has sufficiently advanced to permit the statement, "We understand this now."

A physical theory, in the narrow sense of the word, is a logical structure based on assumptions and definitions which permits one to predict the outcome of a maximum number of different experiments on the basis of a minimum number of postulates. One usually desires the postulates (or axioms) to be as self-evident as possible; one demands simplicity, beauty, and even elegance.

The "maximum number of different experiments" is better expressed as "maximum domain of validity." This, of course, contains the admission that *the validity domain of a theory is limited*, as is obvious to the physicist who has become acquainted with such theories as thermodynamics, geometrical optics, Newtonian mechanics, or Maxwell's electrodynamics.

We shall return to the question of validity limits in Section 1-2. Now we shall assume that we stay within these limits and that we have such a theory. We can then *explain* all phenomena that fall within the validity limits. This means that we are able to account for any such particular happening on the basis of some principles or postulates which are basic to the theory. For example, when two containers of the same gas at different temperatures are suitably connected, a certain average temperature will establish itself; the time-reversed process does not take place. This is explained by means of the second law of thermodynamics. We are satisfied with this explanation and we do not question the second law since it is an axiom on which thermodynamics is built.

Two remarks are essential in this connection. First, the validity limits are not drawn *ad hoc.* When a particular phenomenon is not accounted for by the theory, we cannot gerrymander the validity limits of the theory in order to "save the theory." At some point in the development these limits are established once and for all and cannot be moved arbitrarily. This point of development is reached when the covering theory has been developed, i.e. when the next higher level of theory is invented which contains the first one as an approximation. We shall see this in detail in the next section.

The second remark is the observation that the explanation achieved by the theory is in a certain sense tautological. For how was the theory obtained in the first place? Let us try to reconstruct this at least in a very cursory fashion: Experimental data are studied and compared; regularities are found; new experiments are suggested. Eventually one discovers a "law of nature." This means that one has *invented a proposition* which is confirmed whenever relevant experiments are carried out and for which there does not exist even a single violation. This law was inferred from the results of observations. By its validity we mean that we can safely turn things around, put the law first and deduce from it how the physical system "should behave;" the *prediction* will then be confirmed. This process is possible when nature continues to behave in the same way as it did before (a silent assumption underlying all of science) and when the law is indeed valid.

As we progress in discovering laws of nature (Faraday's law, Ampere's law) we attempt to establish logical relationships between them (which is usually done by means of mathematics) and we hope to arrive eventually at a *deductive logical scheme* (Maxwell's theory of electricity and magnetism) where all the specific instances can be deduced from a small number of fundamental postulates (Maxwell's equations). In this way a theory can be found.

Having gone through all the trouble that is necessary to find the "right" laws and the "right" theory, no one should be surprised that the predictions of the theory indeed agree with experiment or, for that matter, that a given phenomenon can be fully accounted for by the theory. Nevertheless, we say that the theory *explains* the phenomenon. In this sense *scientific explanation* is circular. The emphasis should really be on the existence and correctness of the deductive system, not on the explanation, for the theory is *not derived* or derivable from experiments. It is a mental step into the abstract and general, postulating validity in the future and for all experiments including those that have never been carried out before. The *existence* of a valid theory is therefore the nontrivial and indeed the very remarkable facet of scientific explanation.

We can now define the aim of theoretical physics not as an attempt at explaining the phenomena, but as a striving for the construction of more and more inclusive physical theories and the exploration of their ramifications.

The axiomatization of a theory is usually left to the philosophers of science. Few physicists take interest in this process beyond the purely mathematical aspects of it. Nevertheless, it should be realized that from the logical point of view the final product is an *axiomatic, deductive, logical-mathematical system.*

1-2 THE HIERARCHY OF THEORIES

Every theory is phenomenological to some extent. This means that every theory contains certain quantities which are not determinable by it and which must be fed into it on the basis of appropriate measurements. In optics the index of refraction must be given. In electrodynamics the relationship between $\mathbf{D}$ and $\mathbf{E}$ and between $\mathbf{B}$ and $\mathbf{H}$ must be known; if these relations are linear, the dielectric constant ϵ and the permeability μ must be given. In the theory of charged particles, which we shall discuss in the following chapters, the mass and charge of each particle must be given.

As our knowledge of the physical world increases, we are able to construct theories which are more and more sophisticated and which require fewer and fewer given data. The theories become less phenomenological. For example, solid state physics enables us to *compute* the dielectric constant on the basis of the chemical constituents and the physical conditions of a solid. Progress in theoretical physics, then, clearly points toward the eventual construction of an all-inclusive theory which is phenomenologically minimal. This minimal property is given by the minimum number of units (dimensions) that are necessary: the units of length, time, and mass, or any other three independent ones constructed from them. For a theory can give only dimensionless numbers; and these must be constructed from the quantities *with* dimensions which are fed into it.

A physical theory seems to have only a finite lifetime. Newtonian mechanics was replaced by relativistic mechanics, thermodynamics by statistical mechanics, classical by quantum mechanics. But did these theories become *wrong* after having been proved correct over such a long time? What actually happened was that they continued to provide the correct predictions, but only for a limited set of phenomena. For example, Newtonian mechanics became restricted to phenomena in which the velocities are small compared with the velocity of light. It became an *approximate theory*. Whether the approximate nature becomes apparent in a given instance depends entirely on the accuracy of the experiment.

Thus we learn that an established theory in a certain sense never becomes wrong; it only becomes restricted to a *domain of validity*. This domain of validity is characterized by *inequalities*, for example $(v/c)^2 \ll 1$. This means that if one carries out a measurement with a finite accuracy, δ, as one always does, the range of phenomena correctly described by the theory is clearly determined, e.g. all those which involve velocities such that $(v/c)^2 < \delta$. Improved accuracy, however, may give evidence of the approximate nature of the theory.

But there is another aspect to a theory. While the predictions of a theory will always remain correct when used in the validity domain appropriate to the given measurement, the foundations of the theory, its axioms and the underlying picture (model) may be radically modified by a more general theory: the notions of absolute space and time in Newtonian mechanics are abandoned in the special theory of relativity. But, since the predictions of Newtonian mechanics (within their validity limits) are also the predictions of relativistic mechanics, we can deduce the same results from *either* conceptual framework. The relativistic one, however, is more general and is therefore preferred. It supersedes the Newtonian. In this way, *the conceptual framework of every theory is eventually superseded.*

We can call a theory that agrees with another in all predictions within the latter's validity limits a *covering theory*. Special relativity is a covering theory of Newtonian mechanics. General relativity is a covering theory of special relativity as well as of Newtonian mechanics and Newtonian gravitation theory. Similarly, Maxwell's electrodynamics covers geometrical optics and physical optics; statistical mechanics covers thermodynamics and kinetic theory; and quantum mechanics is the covering theory for classical mechanics.

It is now clear why the validity limits of a theory are in general not known (apart from experimental indications in certain instances) until the covering theory is understood. The long fight between corpuscular optics and wave optics was, to a large extent, due to the lack of knowledge of the appropriate validity limits.

We also observe that there exist several *levels* of theory, one being the covering theory of the next. The highest-level theory is the most general one. The development of physical theory thus builds a *hierarchy* of theories: Newtonian mechanics—special relativity—general relativity, nonrelativistic classical mechanics—nonrelativistic quantum mechanics—relativistic quantum mechanics—relativistic quantum field theory.

This hierarchy attains special significance in view of our human limitations in studying the world around us. We are restricted to a very limited domain of the macroscopic and nonrelativistic classical world in which we function. All observations of nature must be reduced to this "approximation" of nature in order to be perceptible to us: the most complicated studies of nuclear structure, say, are eventually reduced to the macroscopic motion of a hand on a measuring instrument, to a humanly audible click, or to a light flash that is in the very narrow spectral region which we can see. This "translation" from the microscopic, relativistic, or quantum mechanical world to our human one *presupposes the interrelationship of the above hierarchies.*

As a particularly important example, consider quantum mechanics. This theory is meaningless unless it is accessible to observation and measurement. But any measurement involves the interaction of a quantum mechanical system with a classical one (our measuring instrument). In other words, we are at the classical, nonrelativistic end of a system which *must be* described *only partially* by quantum mechanics and *partially* by classical mechanics. This situation makes quantum mechanics incomplete without its classical approximation: the lower-level theory must be contained in the higher-level one as a suitable approximation in order that we can carry out the necessary measurements of the higher-level theory.

It is exactly this situation which permits us, as human beings, to study nature despite the fact that we are part of it.

There is a logical order in theoretical physics. Indeed, such an order is necessary if the endeavor of theoretical physics is to be meaningful and is to succeed in the direction of more and more general axiomatic deductive systems. With this in mind, it is obvious that the study of the relationship between two successive levels of theory, a theory and its minimal covering theory, is of great importance. It provides for the *logical coherence* which exists in theoretical physics but is so often ignored.

Clearly, a higher-level theory can never be derived from a lower-level one. It requires an additional act of invention to produce it. Conversely, it is essential that the lower-level theory be derivable from the covering theory. This must be true not so much with respect to the axiomatic framework, which is in general not a special case of the framework of the covering theory, but with respect to certain basic equations and postulates which contain all the predictive power of the lower-level theory.

The principle of equivalence contains the relationship between general and special relativity.* The correspondence principle dominates the relationship between quantum and classical mechanics. The history of these two principles indicates the importance of the study of the relationship between theories. It shows how one can be guided by such considerations toward a guess of a yet unknown covering theory. The principle of equivalence offers a particularly lucid example of this.

The relationship between classical and quantum mechanics is especially interesting. Here the conceptual frameworks are so radically different that they even employ a different logic. The Aristotelian logic, which pervades all of classical (i.e. nonquantum) physics, is replaced by a quantum logic, which appears strange to our common sense but can be clearly defined mathematically. As a consequence of this switch in logic, and especially as a consequence of the

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* It can be so defined, however, that it contains more than that (cf. Part C of Chapter 3).

widespread lack of recognition of this fact, the beginner in quantum mechanics has considerable difficulties. Paradoxes arise easily when one uses common sense (i.e. Aristotelian logic, trained on the classical physics of our surroundings) to attack quantum mechanics.

1–3 A PLAN FOR THE PRESENTATION OF THE CLASSICAL THEORY OF CHARGED PARTICLES

The presentation of a physical theory should also place the theory in relation to other existing theories, indicate its derivation from the covering theories (if any) in the sense of the foregoing section, and derive from it lower-level theories (if any).

The classical theory of charged particles involves the special theory of relativity and, correspondingly, will be presented in a covariant formulation. It is therefore necessary to start with the basic ideas that enter into the make-up of relativistic theories, emphasizing those aspects which have direct bearing on our theory. This is done in Chapter 3. This chapter also contains a presentation of the foundations of relativistic particle mechanics.

The theory of charged particles is thus a relativistic theory in the sense of special relativity. It is built on two pillars, relativistic particle mechanics and Maxwell-Lorentz electrodynamics. A full chapter is therefore devoted to the latter also (Chapter 4). It is partly an application of the various concepts of special relativity discussed in Chapter 3. For example, the Noether theorem applied to Lorentz invariance is used here for the Maxwell-Lorentz Lagrangian.

Radiation is of special importance; already in classical physics has it an autonomous nature, but quantum physics crowns it by the existence of photons. Therefore, this subject matter is separated from the treatment in Chapter 4, and the following chapter is devoted to it (Chapter 5).

After introducing the two building blocks, the theory of charged particles proper is studied in Chapter 6 for systems of a single charged particle. Since the mathematically consistent formulation of this theory is not generally known, this chapter proceeds in an inductive way. First the usual difficulties are exhibited and the customary formulations are given which lead to the divergences of self-energy and self-stress; as one progresses, more and more of the difficulties disappear until a finite formulation is presented in Section 6–9. The last part of Chapter 6 is devoted to those applications that are of fundamental importance for the theory or exhibit particular techniques for solving the equations of motion.

In Chapter 7 the one-particle theory is generalized to a theory of any finite number of particles. Another generalization is that of a point charge to a finite charge distribution.

Having thus presented the theory, we turn to its relation to other levels of theory (Chapter 8). The nonrelativistic approximation is obtained as a special case, and so is the limit to a neutral particle. The relation to the classical covering theory (general relativity) is also indicated. It is clear that the considerations of Chapter 3 are relevant here and play an important role in the relation of this theory to its covering theory.

The second half of Chapter 8 is devoted to the connection of the classical charged particle theory with quantum physics. Here the validity limits of the classical theory come to the fore, and we discuss the question to what extent a classical description of charged elementary particles is meaningful. The insistence on classical particles without structure (point charges) in Chapter 6 is here justified by the classical approximation (limit) of the quantum mechanical covering theory.

The last chapter serves partly as a review and partly as a means of emphasizing the structure of the theory, its deductive nature, and its physical contents. No attempt is made at an axiomatization.

The theory of charged particles had a long and varied history in the last three quarters of a century. I believe that a full appreciation of it cannot be gained without some knowledge of its development. The following historical chapter therefore precedes the presentation of the theory.